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О СЛЕДАХ В НЕКОТОРЫХ НОВЫХ АНАЛИТИЧЕСКИХ ПРОСТРАНСТВАХ ТИПА ГЕРЦА В ОБЛАСТЯХ ЗИГЕЛЯ В $\mathbb{C}^n$

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ON SHARP TRACES OF SOME NEW ANALYTIC HERZ-TYPE SPACES IN SIEGEL DOMAINS IN $\mathbb{C}^n$

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Получена полная характеризация следов в некоторых новых аналитических пространствах типа Герца в поликруге, единичном шаре, в трубчатых областях над симметрическими конусами и в ограниченных псевдовыпуклых областях с гладкой границей при естественном дополнительном условии на ядро. Наши результаты расширяют известные ранее утверждения.

Ключевые слова: *аналитические функции*, *пространства типа Герца*, *поликруг*, *трубчатые области*, *псевдовыпуклые области.*

We provide complete characterizations of traces of some new analytic spaces of Herz-type in polydisk, unit ball and tubular domains over symmetric cones and bounded pseudoconvex domains with smooth boundary under additional natural condition on kernel. Our results extend previously known assertions.

Keywords: *analytic functions*, *Herz-type spaces*, *polydisk*, *tubular domains*, *pseudoconvex domains.*

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Introduction

This paper is a continuation of a long series of papers of the first author on traces in analytic function spaces on product domains (see [19]–[24], [12] and various references there). The intention of this note to provide new sharp results on traces in some new analytic Herz-type spaces in some general domains (typical Siegel domains of the second type) in $\mathbb{C}^n$. These spaces serve as very natural generalizations of classical Bergman spaces. Note that some new Herz-type spaces of another type where studied recently in papers of the first author [22]–[24]. Hence this note can be considered as continuation of our research on analytic Herz-type spaces. In this paper we at the same time generalize our previous results (see [22]–[24]) obtained for classical Bergman spaces in typical Siegel domains (tubular domains over symmetric cones and bounded pseudoconvex domains with smooth boundary). To be more precise some of our results can be seen in our previous papers in particular case of parameters (putting in all our theorems $p=q$ we get classical Bergman spaces case). We refer the reader to our papers [22]–[24] for new sharp trace theorems in Bergman classes.

In this paper we first consider the case of the unit polydisk (a model case of analytic polyhedron), then the unit ball (a model case of bounded strongly pseudoconvex domains in $\mathbb{C}^n$), and then the bounded strongly pseudoconvex domains with smooth boundary and finally add some vital comments on various other domains in $\mathbb{C}^n$ (bounded symmetric domains and more general minimal bounded homogeneous domains in higher dimension).

We also turn to unbounded Siegel domains, namely general tubular domains T_Ω over symmetric cones in $\mathbb{C}^n$ (a model case is a so-called Lorentz cone). All our proofs of our sharp theorems on traces are mainly based on properties of recently invented so-called r-lattices of tubular domains over symmetric cones and r-lattices of bounded strongly pseudoconvex domains with smooth boundary in $\mathbb{C}^n$ (see [1], [2], [7], [17]).

1 Preliminary results

We start this paper with the simple case of the unit disk and the polydomain to show the core of our constructions first in this case in details. Then the same proof will be repeated for other domains. We need some definitions.

Let D be bounded domains in $\mathbb{C}^n$. Let $H(D)$ be the space of all analytic functions in D. Let $D^m = D\times\ldots\times D$ be polydisk. Let $H(D^m)$ be the space of all analytic functions in D^m, $m\in\mathbb{N}$.

Let

$$A^p_\alpha(D) = \Big\{ f\in H(D) : \|f\|^p_{p,\alpha} =$$

$$= \int_D |f(z)|^p \cdot dist(z,\partial D)^\alpha d\widetilde{m}_2(z) < \infty \Big\}$$

$\alpha > -1$, $0 < p < \infty$ be as usual the Bergman class $d\widetilde{m}_2$ as usual is the Lebesque measure in D. In a standard manner increasing the number of variables we define the spaces of analytic functions of Bergman type in the unit polydomain. Those spaces are Banach spaces for $1 \le p < \infty$ and complete metric spaces for other values of p. Denote by $dV(z)$ or $d\nu(z)$ the Lebeques measure on tube and pseudoconvex domains. We can denote similarly Bergman class in tube and pseudoconvex domains (see [1], [2], [23], [24]). Then we define new natural extentions of Bergman type spaces in the polydisk, namely new analytic Herz type spaces in the unit polydisk. Their complete analogues in other domains will be defined in the next part of this paper. The existence of r-lattices in each domain is crucial for definitions of our new Herz-type spaces. Let

$$H^{p,q}_{\vec{\alpha}}(\tilde{D}^m) = \left\{ f \in H(\tilde{D}^m) : \sum_{k_1=0}^{\infty} \dots \sum_{k_m=0}^{\infty} \sum_{j_1=-2^{k_1}}^{2^{k_1}-1} \dots \sum_{j_m=-2^{k_m}}^{2^{k_m}-1} \times \right.$$

$$\times \left(\int_{\Delta_{k_1,j_1}} \dots \int_{\Delta_{k_m,j_m}} |f(z_1 \dots z_m)|^p \times \right.$$

$$\left. \left. \times \prod_{j=1}^m (1-|z_j|)^{\alpha_j} dm_2(z_j) \right)^{\frac{q}{p}} < \infty \right\},$$

where $\alpha_j > -1$, $j = 1,\dots,m$, $0 < p,q < \infty$, and

$$\Delta_{k,j} = \left\{ z \in \widetilde{D} : z = r\zeta : r \in \left(1 - \tfrac{1}{2^k}, 1 - \tfrac{1}{2^{k+1}}\right], \right.$$

$$\left. \zeta \in \left(\tfrac{-\pi j}{2^k}, \tfrac{\pi(j+1)}{2^k}\right] \ k = 0,1,2,\dots, \quad j = -2^k,\dots,2^k - 1 \right\}.$$

Note if $p = q$ we have classical Bergman space (the same happens in tube and pseudoconvex domains). These are analytic Herz-type spaces in the polydisk constructed based on well-known r-lattices in the unit disc $\widetilde{D}$ (see for these lattices [9] and various referenes there).

The intention of this paper (for the polydisk case) to show that if $f \in H^{p,q}_{\vec{\alpha}}(\widetilde{D}^m)$ then $f(z,\dots,z) \in A^s_\beta(\widetilde{D})$ for some s, β and if $g \in A^s_\beta(\widetilde{D})$, then there is a function f, $f \in H^{p,q}_{\vec{\alpha}}(\widetilde{D}^m)$, so that we have $f(z,\dots,z) = g(z)$, $z \in \widetilde{D}$. (We will put some restrictions on p and q).

This scheme (formulation of problem and even the proof of theorem 2.1 in the polydisk) then will be spread to more complicated domains such us tubular domains over symmetric cones and bounded strongly pseudoconvex domains with smooth boundary.

We alert the reader in advance that since all proofs of all cases are very similar to each other we provide only formulations of the last theorems leaving details of proofs to interested readers, but the proof of the simplest case will be given in this paper with all the details. Moreover we add all comments which are needed to apply the simple case to more complicated cases, all our proofs are based only on several tools and such an approach from our point of view is fully justified. For formulation of our theorems and proofs we will need basic facts on bounded strongly pseudoconvex domains with smooth boundary and on tubular domains over symmetric cones from [7], [17] and papers [24], [23], [15], [26]. For definition of determinant Δ^α and Δ_α function and g_0 and g_0^* type symbols we refer the reader to [7], [17] and to papers [24], [23], [15], [26].

***Lemma* 1.1** [15].

1) *The integral*

$$J_\alpha(y) = \int_{\mathbb{R}^n} \left| \Delta^{-\alpha}\left(\frac{x+iy}{i}\right) \right| dx$$

converges if and only if $\alpha > 2\frac{n}{r} - 1$. *In that case*

$$J_\alpha(y) = \widetilde{C_\alpha} \Delta^{-\alpha + n/r}(y), \ \alpha \in R, \ y \in \Omega.$$

2) *Let* $\alpha \in \mathbb{C}^r$ *and* $y \in \Omega$. *For any multi-indices* s *and* β *and* $t \in \Omega$ *the function*

$$y \mapsto \Delta_\beta(y+t)\Delta_s(y)$$

belongs to $L^1(\Omega, \frac{dy}{\Delta^{n/r}(y)})$ *if and only if* $\Re s > g_0$ *and* $\Re(s+\beta) < -g_0^*$. *In that case we have*

$$\int_\Omega \Delta_\beta(y+t)\Delta_s(y) \frac{dy}{\Delta^{n/r}(y)} = \widetilde{C}_{\beta,s} \Delta_{s+\beta}(t).$$

The following lemma is a complete analogue of Forelly – Rudin type estimates in the tubular domains (see [15]), for classical B_α Bergman kernels (see [23], [24]).

***Lemma* 1.2** [15].

$$\int_{T_\Omega} \Delta^\beta(y) |B_{\alpha+\beta+\frac{n}{r}}(z,w)| dV(z) \le C\Delta^{-\alpha}(v),$$

$\beta > -1$, $\alpha > \frac{n}{r} - 1$, $z = x + iy$, $w = u + iv$.

In next lemmas we provide some basic facts from tubular domains function theory (see, for example, [7], [17] and papers [24], [23], [15], [26]).

The complete analogues of both lemmas 1.3 and 1.4 as well as lemmas 1.5 and 1.6 below are also valid in the polydisck, unit disk, and ball and pseudoconvex domains. Let T_Ω be tube domain.

***Lemma* 1.3** [15]. *Given* $\delta \in (0;1)$ *there exists a sequence of points* $\{z_j\}$ *in* T_Ω *called* δ*-lattice such that calling* $\{B_j\}$ *and* $\{B_j\}$ *the Bergman balls with center* z_j *and radius* δ *and* $\delta/2$ *respectively then*

A) *the balls* $\{B_j\}$ *are pairwise disjoint*;

B) *the balls* $\{B_j\}$ *cover* T_Ω *with finite overlapping*;

C) $$\int_{B_j(z_j,\delta)} \Delta^s(y)dV(z) \asymp$$

$$\asymp \int_{B_j(z_j,\delta)} \Delta^s(y)dV(z) = \widetilde{C_\delta}\Delta^{2\frac{n}{r}+s}(Imz_j);$$

$$s > \tfrac{n}{r}-1, J = |B_\delta(z_j)| \asymp \Delta^{\frac{2n}{r}}(Imz_j), j = 1,\dots,m,$$

$$J \asymp \Delta^{\frac{2n}{r}}(Imw), w \in B_\delta(z_j).$$

***Definition* 1.1.** *Let* μ *be a positive Borel measure in unbounded D domain,* $0 < p,\ q < \infty,\ s > -1.$ *Fix* $r \in (0;\infty)$ *and an* r*-lattice* $\{a_k\}_{k=1}^\infty$. *The analytic space* $A(p,q,d\mu)$ *is the space of all holomorphic functions f such that*

$$\|f\|^q_{A(p,q,d\mu)} = \sum_{k=1}^\infty \left(\int_{B(a_k,r)} |f(z)|^p\, d\mu(z) \right)^{\frac{q}{p}} < \infty.$$

If $d\mu(z) = \Delta^s(Imz)dV(z)$ then we will denote by $A(p,q,s)$ the $A(p,q,d\mu)$. This is Banach space for $\min(p,q) \ge 1$. It is clear that if $p = q$ then we have standart classical Bergman class $A(p,p,s) = A^p_s$.

***Lemma* 1.4** [15]. *For any* $f \in A^2_\nu(T_\Omega)$ *we have for any* $\mu > \frac{2n}{r}-1$

$$f(z) = \int_{T_\Omega} B_\mu(z,w)f(w)\Delta^{\mu-\frac{2n}{r}}(Imw)dV(w).$$

***Lemma* 1.5** [15]. *Let* $\nu > \frac{n}{r}-1,\ \alpha > \frac{n}{r}-1,$ *then for all functions from* A^∞_α *the integral representations of Bergman with Bergman kernel* $B_{\alpha+\nu}(z,w)$ (*with* $\alpha+\nu$ *index*) *is valid.*

***Lemma* 1.6** [15]. *For all* $1 < p < \infty$ *and for all* $\frac{n}{r} \le p_1$, *where* $\frac{1}{p_1}+\frac{1}{p}=1$ *and* $\frac{n}{r}-1 < \nu$ *and for all functions f from* A^p_ν *and for all* $\frac{n}{r}-1 < \alpha$ *the Bergman representation formula with* α *index or with the Bergman kernel* $B_\alpha(z,w)$ *is valid.*

Now we provide the basic facts on strongly pseudoconvex domains taken from [1] and [2]. We denote by ν the normalized Lebegues measure on such domain, $B_D(z,r)$ is Kobayashi ball in pseudoconvex domain with smooth boundary. For definition of classical Bergman spaces in these type domains we refer the reader tothe same papers.

***Lemma* 1.7** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a strongly pseudoconvex bounded domain. Then there exist* $c_1 > 0$ *and, for each* $r \in (0;1)$, *a* $C_{1,r} > 0$ *depending on* r *such that*

$$c_1 r^{2n} d(z_0,\partial D)^{n+1} \le \nu(B_D(z_0,r)) \le C_{1,r} d(z_0,\partial D)^{n+1}$$

for every $z_0 \in D$ *and* $r \in (0,1)$.

***Lemma* 1.8** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded strongly pseudoconvex domain, and* $r \in (0,1)$. *Then*

$$\nu(B_D(\cdot,r)) \approx \delta^{n+1},$$

where the constant depends on r.

***Lemma* 1.9** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded strongly pseudoconvex domain. Then there is* $C > 0$ *such that*

$$\frac{C}{1-r}\delta(z_0) \ge \delta(z) \ge \frac{1-r}{C}\delta(z_0)$$

for all $r \in (0,1), z_0 \in D$ *and* $z \in B_D(z_0,r)$.

***Definition* 1.2.** *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded domain, and* $r > 0$. *An r-lattice in D is a sequence* $\{a_k\} \subset D$ *such that* $D = \bigcup_k B_D(a_k,r)$ *and there exists* $m > 0$ *such that any point in D belongs to at most m balls of the form* $B_D(a_k,R)$, *where* $R = \frac{1}{2}(1+r)$.

The existence of *r*-lattices in bounded strongly pseudoconvex domains with smooth boundary is ensured by the following.

***Lemma* 1.10** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded strongly pseudoconvex domain. Then for every* $r \in (0,1)$ *there exists an r-lattice in* D, *that is there exist* $m \in N$ *and a sequence* $\{a_k\} \subset D$ *of points such that* $D = \bigcup_{k=0}^\infty B_D(a_k,r)$ *and no point of* D *belongs to more than* m *of the balls* $B_D(a_k,R)$, *where* $R = \frac{1}{2}(1+r)$. *Note by lemma 1.8 and 1.9 we have* $\nu_\alpha(B_D(a_k,R)) = (\delta^\alpha(a_k))\nu(B_D(a_k,R)),\ \alpha > -1.$

Sometimes we will call by *r*-lattice the family of Kobayashi balls $B_D(a_k,r)$. Dealing with K unweighted Bergman kernel in bounded pseudoconvex domains with smooth boundary we know $|K(z,a_k)| \asymp |K(a_k,a_k)|$ for any $z \in B_D(a_k,r)$, $r \in (0;1)$ (see, for example, [1], [2] and various references there). This properity plays a vital role in various proofs, though it is well known in the unit ball case (see [29]). It is also valid for weighted K_m Bergman kernels, where $m = (n+1)l,\ l \in N,$ this follows directly from definition of weighted Bergman kernels via Henkin – Ramirez function (see [24], [6], [5], and various references there for this well-known definition). To prove our theorem in context of pseudoconvex domains we will need a stronger condition on weighted Bergman kernel (it is valid in the unit ball [29]). Namely we assume that for t type kernels $|K_t(z,\omega)| \asymp |K_t(a_k,\omega)|$ for any $z \in B_D(a_k,r), r \in (0;1)$ for any ω from our domains (condition K). This type condition on Bergman kernels in pseudoconvex domains can be seen in many papers (see, for example, [14]). It is known this condition is valid in the ball.

From now we assume this condition is satisfied.

We shall use also a submean estimate for nonnegative plurisubharmonic functions on Kobayashi balls.

***Lemma* 1.11** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded strongly pseudoconvex domain. Given* $r \in (0,1)$, *set* $R = \frac{1}{2}(1+r) \in (0,1)$. *Then there exists a* $C_r > 0$ *depending on r such that* $\forall z_0 \in D, \forall z \in B_D(z_0, r)$

$$\chi(z) \le \frac{C_r}{\nu(B_D(z_0,r))} \int\limits_{B_D(z_0,R)} \chi dV$$

for every nonnegative plurisubharmonic function $\chi : D \to \mathbb{R}^+$.

This estimate is valid also in tube (see [23]).

***Lemma* 1.12** [2]. *Let* $D \subset\subset \mathbb{C}^n$ *be a bounded strongly pseudoconvex domain. Then*

$$\|K(\cdot, z_0)\|_2 = \sqrt{K(z_0,z_0)} \approx \delta^{-\frac{n+1}{2}}(z_0) \text{ and } \|K_{z_0}\|_2 \equiv 1$$

for all $z_0 \in D$.

Let D be bounded domain, we define Herz-type spaces for any such domains with *r*-lattices based on balls $B(a_k, r)$. Such a type of analytic spaces are the main objects of this paper. This definition in a standard manner can be applied in product domain case.

***Definition* 1.3.** *Let* μ *be a positive Borel measure in bounded* D *domain,* $0 < p, \; q < \infty, \; s > -1$. *Fix* $r \in (0;\infty)$ *and an* r*-lattice* $\{a_k\}_{k=1}^{\infty}$. *The analytic space* $A(p,q,d\mu)$ *is the space of all holomorphic* f *functions such that*

$$\|f\|^q_{A(p,q,d\mu)} = \sum_{k=1}^{\infty} \left(\int\limits_{B(a_k,r)} |f(z)|^p \, d\mu(z) \right)^{\frac{q}{p}} < \infty.$$

If $d\mu(z) = \delta^s(z) d\nu(z)$ then we will denote by $A(p,q,s)$ the $A(p,q,d\mu)$ spase. This is Banach space for $\min(p,q) \ge 1$. It is clear that if $p = q$ then we have standart classical Bergman class $A(p,p,s) = A^p_s$. These are main objects (spaces) of study of this paper.

2 Main theorems

This section is devoted to formulations and proofs of our main results. We define Herz-type spaces in tubular domains over symmetric cones and bounded strongly pseudoconvex domains with smooth boundary, based on the same idea provided in the unit disk in previous section, then we formulate our new sharp theorems first in the polydisk, then in tubular domains over symmetric cones, then in bounded strongly pseudoconvex domains with smooth boundary in $\mathbb{C}^n$. Proofs of all three assertions are completely parallell. We provide the complete proof of the polydisk case in this paper and add remarks on how to modify the proof for tube and pseudoconvex domains. Then finally some other cases of domains will be shorty discussed by us at the end of the paper.

We define Herz-type spaces in products of tubular domains T_Ω and products of pseudoconvex domains Ω as follows. Note here dealing with weights and measures it is clear from the context we consider products of weights and products of measures, and integration on product domains, though it is not indicated below (see in previous section the definition of parallel analytic Herz type spaces in the polydisk and also for example, see [19]–[21]). Let

$$H^{p,q}_\alpha(T^m_\Omega) = \left\{ f \in H(T^m_\Omega) : \sum_{k=1}^{\infty} \left(\int\limits_{B_T(z_k,r)} |f(\omega)|^{p\alpha} \, (Im\omega) dV(\omega) \right)^{\frac{q}{p}} < \infty \right\};$$

$$H^{p,q}_\alpha(\Omega^m) = \left\{ f \in H(\Omega^m) : \sum_{k=1}^{\infty} \left(\int\limits_{B_\Omega(\widetilde{z_k},r)} |f(\omega)|^p \, (\delta(\omega))^\alpha dV(\omega) \right)^{\frac{q}{p}} < \infty \right\},$$

where $\delta(z) = dist(z, \partial\Omega)$ and where $0 < p, \; q < \infty$, $\alpha > -1$ with usual modification when α is vector. Note for $p = q$ we have classical Bergman space and these are Banach spaces when $\min(q,p) > 1$ and complete metric spaces for other values of parameters and here $\{z^k\} \subset T_\Omega$ and $\{\widetilde{z^k}\} \subset \Omega$ are certain fixed *r*-lattices in tubular domains over symmetric cones T_Ω and in bounded pseudoconvex domains with smooth boundary Ω which were discussed above. Note obviously these analytic spaces in higher dimension depend on $\{z_n\}$ sequences, but we omit this in names of spaces. We denote here by dv(z) or $dV(z)$ the normalized Lebeques measure on products of tubular domains or products of bounded pseudoconvex domain with smooth boundary.

We formulate our theorems in the polydisk, bounded pseudoconvex domains and in tubular domains over symmetric cones then provide detailed proof of first theorem in the polydisk, leaving proofs of theorems 2.2 and 2.3 to readers since the scheme is completelythe same in each case. It is based on properties of *r*-lattices invented recently ([23], [24] for lattices in tube, and [1], [5] for pseudoconvex domains). The simple key idea is the following. Dyadic cubes in the unit disk, and the polydisk and their properties used mainly in the proof of theorem 2.1 should be replaced in the proof of theorem 2.1 below by Kobayashi balls and products of such balls in pseudoconvex domains and by Bergman balls and products of such balls in tubular domains over symmetric cones. The provided lemmas serve as tools which are needed for parallel proofs in these domains. Note various sharp trace theorems in various domains were considered in a long series of papers of the first author (see, for example, [20], [21] and various references there).

We have to define now Bergman type projections in bounded strictly pseudoconvex domain with smooth boundary, and in tubular domains over symmetric cones and in the unit polydisk. Such type operators appeared in trace theorems many times before, see, for example, [19]–[21], [23], [24] and various references there.

In the unit disk we define them as the following operator

$$(P_\alpha f)(\vec{\omega}) = \int_D \frac{f(z)\cdot(1-|z|)^\alpha dm_2(z)}{\prod_{j=1}^{n}(1-\bar{z}\omega_j)^{\frac{\alpha+2}{n}}};$$

$$f \in L^1(D), \vec{\omega} = (\omega_1,\dots,\omega_n),$$

where $\alpha > -1$, $n \in N$. In the tubular domains over symmetric cones we define them as the following Bergman-type integral operator (see for similar operators [15], [23])

$$(P^1_\alpha f)(\vec{z}) = \int_{T_\Omega} \frac{f(\omega)\cdot(\Delta^\alpha(Im\omega))dV(\omega)}{\prod_{j=1}^{m}\Delta^{\frac{\alpha+2\frac{n}{r}}{m}}\left(\frac{z_j-\bar{\omega}}{i}\right)};$$

$$f \in L^1(T_\Omega), \alpha > -1.$$

Finally in the bounded pseudoconvex domains with smooth boundary Ω we define them as

$$(P^2_\alpha f)(\vec{z}) = \int_\Omega f(\omega)\prod_{j=1}^{m} K_\tau(z_j,\omega)\cdot\delta^\alpha(\omega)dV(\omega),$$

$$f \in L^1(\Omega),\ \alpha > -1,\ \tau = \frac{\alpha+n+1}{m}.$$

Such a type of Bergman type integral operators in tube can be seen in [15]. These operators are serving as base of our proofs. Our complete proof of the model case of the polydisk (theorem 2.1) is the systematic use of known properties of dyadic decomposition of the unit disk which can be seen in [9], [20].

To pass this proof to the case of tubular domains over symmetric cone and bounded strongly pseudoconvex domains with smooth boundary (theorems 2.2, 2.3) we have to repeat step by step the same proof replacing on each step properties of *r*-lattices in the disk by properties of r-lattices in tube and pseudoconvex domains (see [1], [2], [23], [24]).

Note in very particular case of unit disk these types of analytic spaces are spaces with quazinorms.

$$\|f\|^p_{A^{p,q}_{\alpha,\beta}} = \int_0^1\left(\int_{|z|<r}|f(z)|^q(1-|z|)^\alpha dm_2(z)\right)^{\frac{p}{q}}(1-r)^\beta dr,$$

where $\alpha > -1$, $\beta > -1$, $0 < p$, $q < \infty$. They were considered previously in papers of M. Jevtic (see [13], see also [12] and references there). These scales of analytic mixed norm spaces are Banach spaces for $\min(p,q) \ge 1$ and complete metric spaces for other values of parameters.

In recent years there was a good amount of activities in both directions in spaces of analytic functions in tubular domains over symmetric cones and pseudoconvex domains with smooth boundary (see, for example, [15] and [2] and various references there also).

***Theorem* 2.1.** *Let* $F \in H^{p,q}_{\vec{\beta}}(\widetilde{D}^m)$, $q \le p$, $0 < p$, $q < \infty$, let β_0 *be large enough. Then* $F(z,\dots,z) \in A^q_{\alpha_1}(\widetilde{D})$ *that is*

$$\int_D |F(z,\dots,z)|^q\ (1-|z|)^{\alpha_1}dm_2(z) < \infty,$$

where

$$\alpha_1 = \sum_{j=1}^{m}(\beta_j+2)\frac{p}{q}-2.$$

Let $f \in A^q_{\alpha_1}(\widetilde{D})$. *Then we can find a function F, so that* $F \in H^{p,q}_{\vec{\beta}}(\widetilde{D}^m)$ *and also* $F(z,\dots,z) = f(z)$, $z \in \widetilde{D}$. *In addition, the Bergman type projection* P_{τ_1} *is mapping from* $A^q_{\alpha_1}(\widetilde{D})$ *to* $H^{p,q}_{\vec{\beta}}(\widetilde{D}^m)$ *as a bounded operator, for all* $\tau_1 > \beta_0$.

Note this type sharp theorem, but for other Herz-type spaces was formulated and proved before in [12] in the unit disk. We formulate below more general theorems of the same type for tubular domains over symmetric cones and bounded pseudoconvex domains with smooth boundary. They have parallel formulations and proofs.

***Theorem* 2.2.** *Let* $F \in H^{p,q}_{\vec{\beta}}(T^m_\Omega)$, $1 < p$, $q < \infty$, $p \ge q$, β_0, α_1 *be large enough. Then* $F(z,\dots,z) \in A^q_{\alpha_1}(T_\Omega)$ *that is*

$$\int_{T_\Omega} |F(z,\dots,z)|^q\ (Imz)^{\alpha_1}dV(z) < \infty,$$

where

$$\alpha_1 = \sum_{j=1}^{m}\left(\beta_j+\frac{2n}{r}\right)\frac{p}{q}-2\frac{n}{r}.$$

Let $f \in A^q_{\alpha_1}(T_\Omega)$. *Then we can find a F function, so that* $F \in H^{p,q}_{\vec{\beta}}(T^m_\Omega)$ *and* $F(z,\dots,z) = f(z)$, $z \in T_\Omega$, *so*

$$TraceH^{p,q}_{\vec{\beta}}(T^m_\Omega) = A^q_{\alpha_1}(T_\Omega).$$

In addition the Bergman type projection $P^1_{\tau_1}$ is mapping from $A^q_{\alpha_1}$ to $H^{p,q}_{\vec{\beta}}$ as a bounded operator for large τ_1, $\tau_1 > \beta_0$.

***Theorem* 2.3.** *Let* $F \in H^{p,q}_{\vec{\beta}}(\Omega^m)$, $0 < p$, $q < \infty$, $n > n_0$, $q \le p$, *and let* β_0, n_0 *be large enough. Then* $F(z,\dots,z) \in A^q_{\alpha_1}(\Omega)$ *that is*

$$\int_{\Omega} |F(z,...,z)|^q \delta(z)^{\alpha_1} dV(z) < \infty,$$

where

$$\alpha_1 = \sum_{j=1}^{m} (\beta_j + n + 1)\frac{p}{q} - (n+1).$$

Let $f \in A^q_{\alpha_1}(\Omega)$. *Then we can find a function F, so that* $F \in H^{p,q}_{\vec{\beta}}(\Omega^m)$ *and* $F(z,...,z) = f(z),\ z \in \Omega$ *so*

$$Trace(H^{p,q}_{\vec{\beta}})(\Omega^m) = (A^q_{\alpha_1})(\Omega),$$

if conditions (*K*) *holds for Bergman kernel of t-type.*

In addition for all $n > n_0$, $\tau_1 > \beta_0$, the Bergman type projection $P^2_{\tau_1}$ is acting from $A^q_{\alpha_1}(\Omega)$ to $H^{p,q}_{\vec{\beta}}(\Omega^m)$ as a bounded operator if the weighted Bergman kernel K_t satisfies (K) condition.

The proof of theorem 2.2 is based on lemmas 1.1–1.6, the proof of theorem 2.3 is based on lemmas 1.7–1.12. Note the complete proof of theorem 2.1 below provides all basic ideas and details needed for proof of theorem 2.2 and 2.3. It can be considered as a proof of theorem 2.3 insimplest case of unit disk.

Remark 2.1. We remark that such type sharp Trace theorems on product domains (tubular domains over symmetric cones and bounded strongly pseudoconvex domains in $\mathbb{C}^n$) where proved recently in [22]–[24]. Note [27] is probably the first paper where product domains were studied. Our results also complement some sharp assertions (sharp trace theorems in the polydisk and unit ball) which can be seen for example in [9], [20], [21]. In [3] we can see several such type results in harmonic function spaces in higher dimension.

Remark 2.2. Note for unit ball B in $\mathbb{C}^n$ these Herz type classes and some of their properties were studied in papers by the author and Songxiao Li (see, for example, [25] and various references there). It was shown in [25] in particular that if μ is a positive Borel measure on B and $\{a_k\}$ is a sampling sequence based on $B(a_k, r)$ Bergman balls $\alpha > -1$, $0 < p_i,\ q_i < \infty,\ f_i \in H(B)$, and $\sum_{i=1}^{n}\left(\frac{1}{q_i}\right) = 1.$

Then

$$\int_D \prod_{k=1}^{n} |f_i(z)|^{p_i}\, d\mu(z) \le$$

$$\le c\prod_{i=1}^{n}\left(\sum_{k=1}^{\infty}\left(\iint_{D(a_k,r)} |f_i(z)|^{p_i}(1-|z|)^{\alpha} dV(z)\right)^{q_i}\right)$$

if and only if $\mu(D(a_k,r)) \le c(1-|a_k|)^{m(n+1+\alpha)}$ for every $k = 1,2,3,...$ Using these Bergman $B(a_k,r)$ balls and their properties which can be seen in [29], [25], a complete copy of theorem 2.1 in the unit ball in $\mathbb{C}^n$ can be also formulated, we leave this procedure to readers.

The proof of theorem 2.1. The proof of theorem 2.1 hinges on lemmas 1.7–1.12 from previous section in case when our pseudoconvex domains is a unit disk.

First note that by lemmas 1.8–1.10 applied in the unit disk if $f \in A^p_\alpha$ then for all $\alpha > -1$, $0 < p < \infty$ we have that (see also [19], [9] for similar arguments); $\widetilde{f(z)} = f(z,...,z)$

$$\int_D |\widetilde{f(z)}|^p (1-|z|)^{\alpha} dm_2(z) =$$

$$= \sum_{j,k}\int_{\Delta_{j,k}} (|\widetilde{f(z)}|^p)(1-|z|)^{\alpha} dm_2(z) \le$$

$$\le c\sum_{j,k}(\max_{\Delta_{jk}})(|\widetilde{f(z)}|^p)(\int_{\Delta_{j,k}}(1-|z|)^{\alpha} dm_2(z)) \le$$

$$\le c\sum_{j,k}(\max_{\Delta_{jk}})|\widetilde{f(z)}|^p\cdot\left(2^{-k\alpha}\cdot 2^{-2k}\right) \le J.$$

Using lemma 1.11 in the polydisk

$$J \le c\sum_{k_1\ge 0}\sum_{k_n\ge 0}\sum_{j_1,...,j_n}\max_{\substack{z_1\in\Delta_{j_1,k_1}\\ z_n\in\Delta_{j_n,k_n}}}(|f(z_1,...,z_n)|^q)^{\frac{p}{q}}\times$$

$$\times(2^{-k_1\frac{(\alpha+2)}{n}}...2^{-k_n\frac{(\alpha+2)}{n}}) \le$$

$$\le c\sum_{k_1\ge 0}\sum_{k_n\ge 0}\sum_{j_1,...,j_n}\left(\int_{\Delta_{j_1,k_1}}...\int_{\Delta_{j_n,k_n}}|f(z_1,...,z_n)|^q\, dm_{2n}(z)\right)^{\frac{p}{q}}\times$$

$$\times\left[2^{-k_1\frac{(\alpha+2)}{n}}...2^{-k_n\frac{(\alpha+2)}{n}}\right]\cdot\left(2^{2k_1\frac{p}{q}}...2^{2k_n\frac{p}{q}}\right) \le$$

$$\le c\sum_{k_1...k_n}\sum_{j_1,...,j_n}\left(\int_{\Delta_{j_1,k_1}}...\int_{\Delta_{j_n,k_n}}|f(z_1,...,z_n)|^q\times\right.$$

$$\left.\times\prod_{j=1}^{n}(1-|z_j|)^{\beta_j}dm_{2n}(z)\right)^{\frac{p}{q}}$$

where $\beta_i = \frac{(\alpha+2)}{n}\frac{q}{p} - 2,\ \ j = 1,...,n,\ \ 0 < p,\ \ q < \infty.$

Note in last estimate we used the fact ([9]) that for subharmonic *f* function in the polydisk

$$\max_{\substack{z_1\in\Delta_{j,k}\\ z_n\in\Delta_{j_n,k_n}}}|f(z_1,...,z_n)|^q \le \frac{c}{m_2(\Delta_{j_1,k_1})...m_2(\Delta_{j_n,k_n})}\times$$

$$\times\int_{\Delta_{j_1,k_1}}...\int_{\Delta_{j_n,k_n}}|f(z_1,...,z_n)|^q\, dm_2(z_1)...dm_2(z_n), \quad (2.1)$$

$m_2(_{jk}) = 2^{-2k}$, $k = 0,1,2,...$ This estimate follows from related estimate of one variable function (lemma 1.11) applied several times by each variable separately. The vital point in the chain of estimates we consider that it is valid also in tubular domains and pseudoconvex domains based fully on properties of r-lattices which we provided in previous section, but with other parameters (see also [24], [23], [5], [2], [1] for similar arguments). Let us show the reverse assertion, that for each analytic function *f*, $f \in A^p_\alpha(D)$ there is a analytic function *F*,

$F(z,...,z)=f(z)$ and $z\in D$, $F\in H_\beta^{p,q}(D^n)$. For this reason we will use the following simple observation related with dyadic cubes $\{\Delta_{jk}\}_{j,k}$ and Bergman kernel $[Q_\beta](z,\omega)=\frac{(1-|z|)^\beta}{(1-\bar{\omega}z)^{\beta+2}}$, $\beta>-1$ in the unit disk where $z\in D$, $\omega\in D$. The importance of simplicity of arguments and estimates below for us again is based on the fact that the same estimates with other values of parameters are valid also in tubular domains and in pseudoconvex domains (based on lemmas related with r-latticesfrom previous section).

Note first that (see also [20], [9]) using (2.1) we have the following estimates in one dimension for the Bergman kernel. For center z_{jk} of "dyadic cube" Δ_{jk} by lemma 1.8–1.10 and 1.11 we have

$$\sum_{k\ge 0}\sum_j \frac{(1-|z_{jk}|)^\beta}{|1-\bar{z}z_{jk}|^\tau}\le \le c\sum_{k\ge 0}\sum_j \int_{\Delta_{jk}} \frac{(1-|\tilde{z}|)^{\beta-2}dm_2(\tilde{z})}{|1-\bar{z}\tilde{z}|^\tau}\le \le \frac{C_1}{(1-|z|)^{\tau-\beta}}, \tag{2.2}$$

$z\in D$, $\tau>0$, $\beta\ge 1$, $\tau>\beta$ we will need also the following estimate (2.2)

$$\int_{\Delta_{jk}} \frac{(1-|z|)^\beta}{|1-\bar{z}\omega|^\tau}dm_2(z)\le c\left(\frac{2^{-k\beta}}{|1-\overline{z_{jk}\omega}|^\tau}\right),\ \omega\in D \tag{2.3}$$

(see, for example, [9] and references there) and

$$\frac{2^{-k\beta}}{|1-\overline{z_{jk}\omega}|^\tau}\le c\frac{(1-|z_{jk}|)^\beta}{|1-z_{jk}\omega|^\tau};$$
$$\tau>0,\beta\ge 0, z_{jk}\in\{\Delta_{jk}\}, \omega\in D,$$

z_{jk} is the center of Δ_{jk}. Note that all three estimates are also valid in tubular domains and pseudoconvex domains based on lemmas 1.2–1.3, 1.5–1.10 and some conditions on Bergman Kernels (see also [24], [23], [2], [1]). The second estimate for tube can be seen in [18]. For pseudoconvex domains it follows from (K) condition. Using these two estimates and well – known properties of dyadic decomposition of the unit disk (dyadic cubes Δ_{jk}), (see, for example, [9], [19]–[21] and references there) we will have the following inequalities in the unit disk D.

First note that if $f\in A_\alpha^p(D)$ then $F\in H(D^m)$, where

$$F(z_1,...,z_n)=\int_D \frac{f(\omega)(1-|\omega|)^\beta dm_2(\omega)}{\prod_{j=1}^n (1-\bar{\omega}z_j)^{\frac{\beta+2}{n}}},$$
$$z_j\in D, j=1,...,n$$

and β is large enough number, $\beta>\beta_0$ then note that for $p\le 1$ and then for $p>1$ we have that

$$|F(z_1,...,z_n)|^p\le \le\int_D \frac{|f(\omega)|^p(1-|\omega|)^{\beta p+2p-2}dm_2(\omega)}{\left|\prod_{j=1}^n (1-\bar{\omega}z_j)^{\frac{\beta+2}{n}p}\right|} \tag{2.4}$$

for $z_j\in D$, $j=1,...,n$ and for $\gamma_1+\gamma_2=\left(\frac{\beta+2}{n}\right)$, $\gamma_2>0$

$$|F(z_1,...,z_n)|^p\le \le\int_D \frac{|f(\omega)|^p(1-|\omega|)^\beta dm_2(\omega)}{\left|\prod_{j=1}^n (1-\bar{\omega}z_j)^{p\gamma_1}\right|}\times \times\prod_{j=1}^n (1-|z_j|)^{\left(\frac{\beta+2}{n}-\gamma_2\right)p-\frac{(\beta+2)}{n}} \tag{2.5}$$

both estimates can be seen in [12]. We must show now $F\in H_\beta^{p,q}(D^m)$ this will finish the proof. Note last two estimates again are valid in tubular domains and pseudoconvex domains ([24], [23], [2], [1]). The last estimate in particular follows from Holders inequality and Forelly – Rudin type estimates directly (see for tube lemma 1.2). The previous one in pseudoconvex domains can be seen in [5], in tube in [24], [23]. This indeed will finish the proof since we have that $F(z,...,z)=f(z)$, $z\in D$ and this will hold since of well-known Bergman representation formula in the unit disk (see, for example, [9], chapter 4 and references there). The same formula is valid in tube and pseudoconvex domains (see lemmas 1.4, 1.5, 1.6 for tube case).

From (2.4) and (2.5) using (2.2) and (2.3) we will have that the following estimates are true. We use that β is large enough.

$$\sum_{k_1...k_n}\sum_{j_1,...,j_n}\left(\int_{\Delta_{j_1,k_1}}...\int_{\Delta_{j_n,k_n}}|F(z_1,...,z_n)|^p\times \times\prod_{j=1}^n(1-|z_j|)^{\beta_j}dm_{2n}(z)\right)^{\frac{q}{p}}\le$$

$$\le c\sum_{k_1...k_n}\sum_{j_1,...,j_n}\left(\int_{\Delta_{j_1,k_1}}... \int_{\Delta_{j_n,k_n}}\left(\int_D\frac{|f(\omega)|^p(1-|\omega|)^{\beta p+2p-2}dm_2(\omega)}{\prod_{j=1}^n|1-\bar{\omega}z_j|^{\frac{\beta+2}{n}p}}\right)\times \times\prod_{j=1}^n(1-|z_j|)^{\beta_j}dm_{2n}(z)\right)^{\frac{q}{p}}\le$$

$$\le c_1\sum_{k_1...k_n}\sum_{j_1,...,j_n}\left(\int_D\frac{|f(\omega)|^p(1-|\omega|)^{\beta p+2p-2}dm_2(\omega)}{\prod_{j=1}^n(|1-\bar{\omega}z_j^*|)^{\frac{\beta+2}{n}p-(\beta_j+2)}}\right)^{\frac{q}{p}}\le$$

$$\le c\int_D \frac{|f(\omega)|^q (1-|\omega|)^{(\beta p+2p-2)\frac{q}{p}+2\frac{q}{p}-2} dm_2(\omega)}{\prod_j^n (1-|\omega|)^{[\frac{\beta+2}{n}p-(\beta_j+2)]\frac{q}{p}}} \le$$

$$\le c\int_D |f(\omega)|^q (1-|\omega|)^{\sum_{j=1}^n(\beta_j+2)\frac{p}{q}-2} dm_2(\omega);$$

here z_s^* is the center of Δ_{j_s,k_s}.

Indeed we have that

$$(2p-2+\beta p)\frac{q}{p}+2\frac{q}{p}-2+$$

$$+\sum_{j=1}^n\left(\frac{\beta+2}{n}(-q)+(\beta_j+2)\frac{q}{p}\right)=$$

$$=(-2)+\sum_{j=1}^n(\beta_j+2)\frac{q}{p}=\tau,\ \tau>-1.$$

Thus our theorem is proved for $q\le p\le 1$ case. Let us assume $q<p,\ \ p>1$. Then we have from estimate (2.5) the following estimate

$$\|f\|^q_{H^{p,q}_\beta}(D)\le$$

$$\le c\sum_{k_1\ldots k_n}\sum_{j_1,\ldots,j_n}\left(\int_D \frac{|f(\omega)|^p (1-|\omega|)^{\beta p+2p-2} dm_2(\omega)}{\prod_j^n (|1-\overline{\omega}z_j^*|)^{\frac{(\beta+2)}{n}p-(\beta_j+2)}}\right)^{\frac{q}{p}}=K.$$

Using (2.5) again for inner integral for $\frac{q}{p}>1$ we have

$$s_1^j+s_2^j=\tau=\frac{(\beta+2)}{n}p-(\beta_j+2),$$

$$s_1^j>0,\ s_2^j>0,\ j=1\ldots n,$$

$$K\le c\sum_{k_1\ldots k_n}\sum_{j_1,\ldots,j_n}\int_D \frac{|f(\omega)|^p (1-|\omega|)^{(2p-2)(\beta p)} dm_2(\omega)}{\prod_{j=1}^n (|1-\overline{\omega}z_j^*|)^{(s_1^j)\frac{q}{p}}}\times$$

$$\times\prod_{S=1}^n (1-|z_s^*|)^{s_1^j\left(\frac{q}{p}\right)-\gamma_1^j};$$

$$\gamma_1^j=\frac{(\beta+2)}{n}p-(\beta_j+2),\ j=1,\ldots,n$$

where z_s^* is a center of Δ_{j_s,k_s}, $s=1,\ldots,n$

$$K\le c\int_D |f(\omega)|^q (1-|\omega|)^\tau dm_2(\omega),$$

$$\tau=\sum_{j=1}^m(\beta_j+2)\frac{p}{q}-2;\tau>-1$$

we used (2.2), (2.3) here again.

We omit easy calculations with indexes here. The crucial fact here is that β is large enough positive number, $\beta>\beta_0$ and hence s_1^j and s_2^j can be chosen as large as we wish together with γ_1^j, for all $j=1,\ldots,n.$ Theorem 2.1 is proved.

Now we turn to the proof of theorems 2.2 and 2.3, noting they are completely parallel to the proof we have just provided above (we refer the reader also to several detailed remarks given within the proof of theorem 2.1).

The main ingredients here are r-lattices and their properties and properties of Whitney type decomposition (Bergman or Kobayashi balls) of domains which we provided above in previous section. We omit details.

Remark 2.3. Trace theorems and related issues concerning Bergman type projection in various general domains were under intensive study during last decades (see, for example, [10]–[21], [24], [23]). Various results in this directions were obtained and even some applications to Hardy type inequalities and duality theorems were found (see, for example, [15]–[17] and various references there). We do not discuss these issues in this note, but hope to return to them in our forthcoming papers.

Remark 2.4. Note that Trace theorems have various applications in function and operator theory (see, for example, [4], [8] and references there). Hence our theorems may also have such a type of applications.

Remark 2.5. Finally we add some analysis concerning other domains and related to these results on them.

Our theorems are also partially valid in bounded symmetric domains, since all machinery we have used is also valid there, starting from Forelly-Rudin type estimates and Whitney type decomposition of such domains. We refer the reader to papers [10], [28] where appropriate machinery can be found, the same can be even applied to more general minimal bounded homogeneous domains in higher dimension and appropriate estimates can be seen in a series of recent papers of S. Yamaji (see, for example, [10], [28], [11] and references there).

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